Convergence of Mimetic Finite Difference Method for Diffusion Problems on Polyhedral Meshes

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In many applications meshes with general type elements are more preferable than standard tetrahedral or hexahedral meshes. Allowing arbitrary shape for a mesh element provides greater flexibility in the mesh generation process, especially in the regions where the geometry is extremely complex. For example, in a reservoir simulation, the thinning or tapering out ("pinching out") of geological layers can be easily modeled with pentahedrons and prisms.

The predictions and the insights gained from numerical simulations on polyhedral meshes are trustworthy only if the reliable and accurate discretization methods have been used. An example of such a method is the mimetic finite difference (MFD) method [1]. For the linear diffusion problem with a sufficiently smooth solution the MFD method exhibits the second-order convergence rate for the fluid pressure and the first-order convergence rate for the fluid velocity.

Similar convergence rates are observed in other lower order discretization methods under different assumptions on the computational mesh (e.g., finite element methods on simplicial meshes). The main advantage of the MFD method is its flexibility. We have proved in [2] that the method converges on meshes with degenerate and nonconvex elements with flat faces (see Fig. 1). The irregular elements appear in mesh refinenement methods (with hanging nodes), in moving mesh methods and in nonmatching mesh methods.

For the linear diffusion problem, the MFD method mimics the Gauss divergence theorem, the symmetry between the gradient and divergence operators, and the null spaces of the involved operators. Therefore, it produces the discretization scheme which is locally conservative, exact for uniform flow, and results in symmetric positive definite coefficient matrix. The above properties are held for the full diffusion tensor.

The original proof of the convergence of the MFD method on quadrilateral and

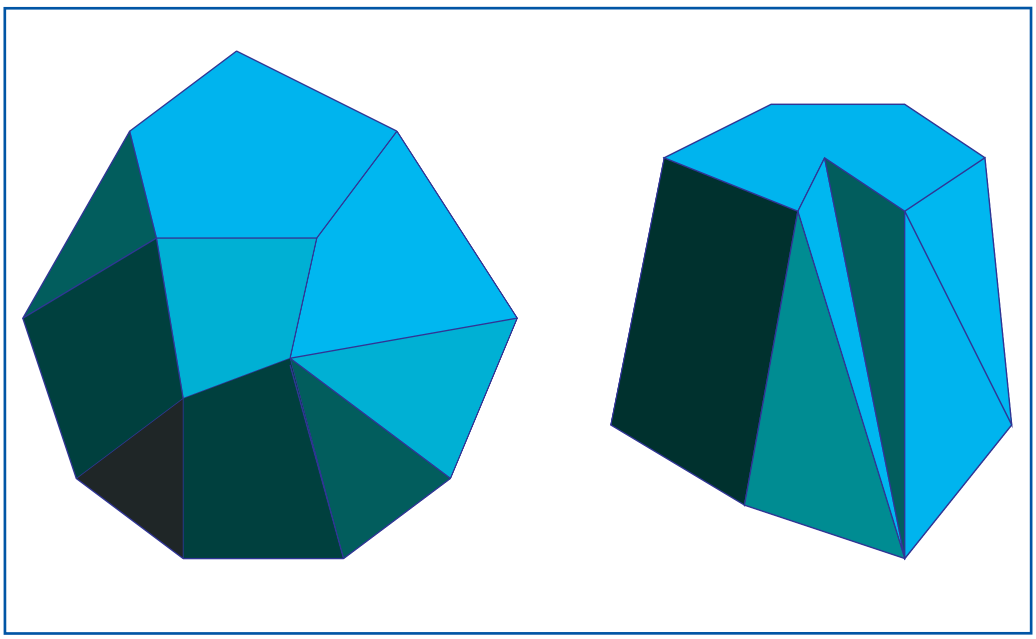

Figure 1— Two acceptable polyhedral elements.

triangular meshes was based on establishing a relationship with a mixed finite element method. In [2] we developed a new methodology that has both theoretical and practical value. It provides a constructive way for extending the MFD method to polyhedral meshes with curved faces. At the moment, a curved face may be approximated by flat triangles, which still gives a discrete problem with smaller number of unknowns relative to a tetrahedral partition.

[1] Y. Kuznetsov, K. Lipnikov, and M. Shashkov, "The Mimetic Finite Difference Method on Polygonal Meshes for Diffusion-type Problems" (to be published in *Comp. Geosciences*, 2004).
[2] F. Brezzi, K. Lipnikov, and M. Shashkov, "Convergence of Mimetic Finite Difference Method for Diffusion Problems on Polyhedral Meshes," Los Alamos National Laboratory report LA-UR-04-5756 (to be published in *SIAM J. Numer. Anal.*, 2005).